Harvard Undergraduate Science Olympiad 2025

Final Round

Physics: 9th-10th Grade
Answer Key

Part A: Multiple Choice

1. A1: a
2. A2: b
3. A3: a
4. A4: c
5. A5: c
6. A6: c
7. A7: b
8. A8: b

9. A9: c

10. A10: d

11. A11: b

12. A12: c

13. A13: b

14. A14: b

15. A15: b

16. A16: b

17. A17: c

18. A18: a

Part B: Free response

Problem B1

Ball A: The horizontal equation for position is $\frac{gt_A^2}{2} = h$, thus we calculate that:

$$t_A = \sqrt{\frac{2h}{g}} = \sqrt{2\frac{1.25m}{10\frac{m}{s^2}}} = 0.5s$$

Since the ball is thrown horizontally, we have that $d_A = v_A \cdot t_A = 5\frac{m}{s} \cdot 0.5s = 2.5m$
Ball B: Its horizontal velocity is $v_{Bh} = v_B \cdot \cos(30°) = 10\frac{m}{s} \cdot \sqrt{3}/2 = 8.66\frac{m}{s}$.
Its vertical velocity is $v_{Bv} = v_B \cdot \sin(30°) = 10\frac{m}{s} \cdot \frac{1}{2} = 5\frac{m}{s}$.
We set up the horizontal equation for position: $h + v_{Bv}t - gt^2/2 = 0$. Using the equation for the solutions to the quadratic equation, we get that:

$$t_B = \frac{2}{g}\left[-v_{Bv} \pm \sqrt{v_{bv}^2 + 4h\frac{g}{2}}\right] = \frac{-1}{10\frac{m}{s^2}}\left[-5\frac{m}{s} \pm \sqrt{25\frac{m^2}{s^2} + 2 \cdot 1.25m \cdot 10\frac{m}{s^2}}\right] = 1.21s$$

We pick the negative sign solution because the time has to be positive. Thus, the displacement for B is $d_B = v_{Bh} \cdot t_B = 8.66\frac{m}{s} \cdot 1.21s = 10.48m$.
Ball C: Since ball C doesn't have any vertical component of the velocity, it will take the same amount of time as ball A to hit the ground and $t_C = t_A = 0.5s$. Its displacement will be $d_C = 0$ since it has no horizontal velocity either.

Comparing the displacements and times we get that $d_C < d_A < d_B$ and that $t_A = t_C < t_B$.

Problem B2 Circuits

a) $\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} \implies R_p = \frac{R_2 R_3}{R_2+R_3} = \frac{3\cdot 6}{9}\Omega = 2\Omega$

Then, $R_3 = R_1 + R_p = 6\Omega + 2\Omega = 8\Omega$

b) $I = \frac{V}{R} = \frac{12V}{8\Omega} = 1.5\ A$

c) $V_1 = IR_1 = 1.5A \cdot 6\Omega = 9V \implies V_{23} = V - V_1 = 3V$

d) $V_{23} = I_2 R_2 \implies I_2 = \frac{V_{23}}{R_2} = \frac{3V}{3\Omega} = 1A$

$V_{23} = I_3 R_3 \implies I_3 = \frac{V_{23}}{R_3} = \frac{3V}{6A} = 0.5A$

Consistency check: $I_1 = I_2 + I_3$

e) $P_2 = I_2^2 R_2 = (1A)^2 \cdot 3\Omega = 3\ W$

f) When R_2 burns down, no current can flow trough it. Then, R_2 and R_3 are in series, and $R'_e = R_1 + R_3 = 12\Omega \implies I' = \frac{V}{R'_e} = \frac{12V}{12\Omega} = 1A$.

Problem B3 Conical pendulum

a) The equilibrium of forces requires:
$T\cos(\alpha) = mg$ (relation 1)
$T\sin(\alpha) = m\omega_0^2 R$ (relation 2)

Dividing relation 2 by relation 1: $\tan(\alpha) = \frac{\omega_0^2 R}{g}$

But $\tan(\alpha)$ can be represented trigonometrically: $\tan(\alpha) = \frac{R}{\sqrt{l^2-R^2}}$. Thus, $\frac{gR}{\sqrt{l^2-R^2}} = \omega_0^2 R \implies l^2 - R^2 = \frac{g^2}{\omega_0^4}$, so $R = \sqrt{l^2 - \frac{g^2}{\omega_0^4}} = 0.79m$.

Note: For the set-up to make physical sense, we need $l^2 - \frac{g^2}{\omega_0^4} > 0 \implies \omega_0 > \sqrt{\frac{g}{l}} = 3.13\frac{rad}{s}$

b) Again, the equilibrium of forces requires:
$F\cos(\alpha_0) = mg$ (relation 1)

$F\sin(\alpha_0) = m\omega_0^2 R$ (relation 2)

$T_0 = \frac{2\pi}{\omega} = \frac{2\pi R}{v}$, so $\omega = \frac{2\pi}{T_0}$

Then relation 2 can be written: $F\sin(\alpha_0) = \frac{m}{R}(\frac{2\pi R}{T_0})^2$.

Dividing relation 2 by relation 1, we get $\tan(\alpha_0) = \frac{4\pi R}{gT_0^2} \implies T_0^2 = \frac{4\pi R}{g\tan(\alpha_0)} \implies T_0 = 2\pi\sqrt{\frac{R}{g\tan(\alpha_0)}} = 2\pi\sqrt{\frac{R\cos(\alpha_0)}{g\sin(\alpha_0)}}$. But $L\sin(\alpha_0) = R \implies \frac{R}{\sin(\alpha_0)} = L \implies T_0 = 2\pi\sqrt{\frac{L\cos(\alpha_0)}{g}} \approx 2\pi\sqrt{\frac{1}{20}}\ s = 1.67\ s$

When the elevator is going down, the equivalent gravitational acceleration felt by the ball is $g' = g - a$, and is pointing downwards. Then, the new oscillation period is:

$$T' = 2\pi\sqrt{\frac{L\cos(\alpha_1)}{g'}} = 2\pi\sqrt{\frac{L\cos(\alpha_1)}{g-a}} = T_0\sqrt{\frac{g\cos(\alpha_1)}{(g-a)\cos(\alpha_0)}} = 1.178\ s$$

Problem B4 Infinite Resistor Ladder

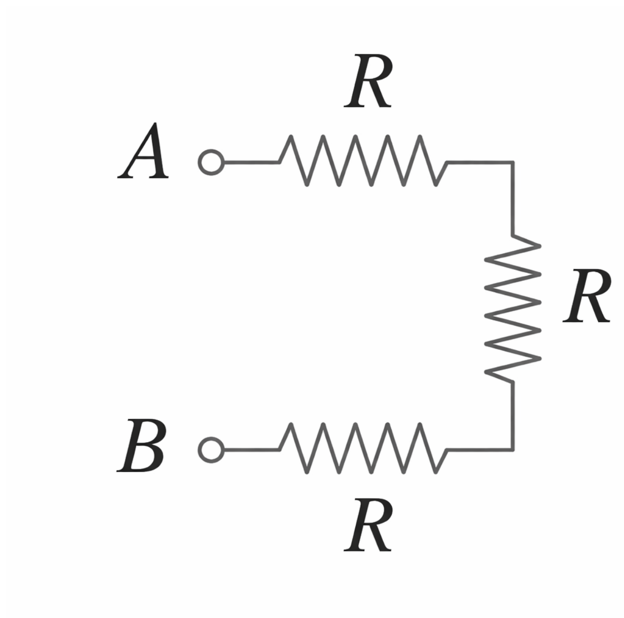

Figure 1: Unit Cell of the infinite resistor ladder

We define the above resistor set-up to be a "cell" of the infinite resistor ladder.

Since the ladder is made up of an infinite number of resistors, and thus of an infinite number of cells, adding a new cell to the set-up does not change the overall resistance of the ladder.

Let R_e be the equivalent resistance of the infinite ladder. Add a new cell to the set-up. Then, $\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R_e} \implies R_p = \frac{RR_e}{R+R_e}$.
Then,

$$R_{AB} = 2R + R_p,$$

so $R_{AB} = 2R + \frac{RR_e}{R+R_e}$.

But $R_{AB} = R_e$. Then, $R_e = R + \frac{RR_e}{R+R_e} \implies RR_e + R_e^2 = 2R^2 + 2RR_e + RR_e$, so the quadratic equation that determines the equivalent resistance of the system is:

$$R_e^2 - 2RR_e - 2R^2 = 0$$

$\implies R_e = \frac{2R \pm \sqrt{4R^2+8R^2}}{2} = \frac{2R \pm R\sqrt{12}}{2} = R(1 \pm \sqrt{3})$. Since $\sqrt{3} \approx 1.73$, the only solution that makes physical sense is $R_e = R(1+\sqrt{3})$.

Problem B5 Around the world

As hinted, consider an infinitesimal mass element dm, placed a distance x away from the rotation axis, and of length dx. The element undergoes a uniform circular motion with angular velocity ω.

Radially, the force equilibrium condition reads:

$$T(x) = T(x) + dT + (dm)\omega^2 x,$$

where $T(x)$ is the internal axial force transmitted across a cross-section of the rod at distance x from the pivot.

Also, $dm = \frac{m}{l} \cdot dx$. Then, $dT = -\frac{m}{l}\omega^2 x \ dx$. Integrating indefinitely, we get

$$T = \int dT = -\int \frac{m}{l}\omega^2 x \ dx = -\frac{m}{l}\omega^2 \int x \ dx = -\frac{m}{l}\omega^2 \frac{x^2}{2} + C$$

The constant of integration is determined by the condition that $T(l) = 0$:

$$0 = -(\frac{m}{l})\omega^2 \frac{l^2}{2} + C$$

hence

$$C = \frac{m}{l}\omega^2\frac{l^2}{2}.$$

Therefore,

$$T(x) = \frac{1}{2l}m\omega^2(l^2 - x^2)$$

b) The reaction force of the hinge on the rod is just $T(0)$:

$$T(0) = -\frac{1}{2}m\omega^2 l$$

Problem B6 Thermodynamics

a) By the ideal gas law, $PV = nRT$. Since the gas is compressed isothermally, $P_1V_1 = P_2V_2$, and $V_2 = \frac{V_1}{2} \implies P_1V_1 = P_2\frac{V_1}{2}$, $P_2 = 2P_1 = 2 \cdot 10^5 Pa$

b) $W_1 = nRT_1 \ln(\frac{V_2}{V_1}) = nRT_1 \ln(\frac{1}{2}) = -nRT_1 \ln(2)$.

Numerically, $W_1 \approx -0.5 \cdot 8.1 \cdot 300 \cdot 0.693\ J = -864\ J$. Thus, the work **done by the gas** is negative, so there is work **done on the gas**.

c) Again, we invoke the ideal gas law: $PV = nRT$. Since the piston is locked, the volume is constant, so the gas undergoes an isochoric process. Then, $\frac{P_2}{T_2} = \frac{P_3}{T_3}$. Then, $P_3 = \frac{T_3P_2}{T_2} = 1.5P_2 = 3 \cdot 10^5 Pa$.

d) The volume of the gas during step 2 is constant, hence the work done by the gas is zero $\implies W_2 = 0$.

e) Since the work done by the gas is zero, the heat added during step 2 is equal to the change in internal energy: $W_2 = \Delta U_2 = nC_{V,molar}(\Delta T)$.

The ideal gas at hand is monatomic, so it has 3 degrees of freedom per atom (each atom can move in the x, y, and z direction, independently). The equipartition theorem states that each quadratic degree of freedom contributes, on average, $\frac{1}{2}k_BT$ per atom to the kinetic energy of the gas. Then, we get $U = N\frac{3}{2}k_BT$.

Using $Nk_B = nR$, we get $U = \frac{3}{2}nRT$. $C_V = (\frac{\partial U}{\partial T})_V = \frac{3}{2}nR$, so $C_{V,molar} = \frac{C_V}{n} = \frac{3}{2}R$, for a monatomic gas.

Thus, we finally get $W_2 = nC_{V,molar}(T_3 - T_2) = 0.5 \cdot 8.31 \cdot 150J \approx 935J$

e) In step 1, the gas undergoes an isothermal process, so $\Delta U_1 = 0 \implies Q_1 = W_1 = -864J$.

In the second step, $Q_2 = U_2 = 935J$. Then, $Q_{net} = Q_1 + Q_2 \approx 71\ J > 0$

Problem B7 Rolling without slipping

Along the incline,

$$mg\sin\theta - F_f = ma.$$

Taking torques about the center of mass,

$$F_f R = I\alpha, \qquad a = \alpha R \;\Rightarrow\; F_f = \frac{Ia}{R^2}.$$

Substitute into the first equation:

$$mg\sin\theta = a\left(m + \frac{I}{R^2}\right) \;\Rightarrow\; a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}.$$

For a solid cylinder $I = \frac{1}{2}mR^2$, so

$$a = \frac{g\sin\theta}{1 + \frac{1}{2}} = \frac{2}{3}g\sin\theta.$$